Numerical integration with Simpson’s rule

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| Overview | Numerical integration is the process of determining the area “under” some function.  Numerical integration calculates this area by dividing it into vertical “strips” and summing their individual areas.  The key is to minimize the error in this approximation. |  |

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| Simpson’s rule | Simpson’s rule can be used to integrate a symmetrical statistical distribution function over a specified range (e.g., from 0 to some value *x*).   1. *num\_seg* = initial number of segments, an even number 2. W = *x*/*num\_seg*, the segment width 3. E = the acceptable error, e.g., 0.0000001 4. Compute the integral value with the following equation.      1. Compute the integral value again, but this time with *num\_seg* = *num\_seg*\*2. 2. If the difference between these two results is greater than E, double *num\_seg* and compute the integral value again. Continue doing this until the difference between the last two results is less than E. The latest result is the answer. |

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Numerical integration with Simpson’s rule, Continued

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| A simple example | Let’s look at a simple function, where F(*x*) = 2*x*.  Note: This example is a triangle. The area of a triangle is | F(*x*) = 2*x*  *num\_seg = 4*  *W = 4/4 = 1*  *x* = 4 |

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|  | In this example, we can expand Simpson’s rule    to    and then substitute calculated values for the function F(*x*)= 2*x* |

The t distribution

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| Overview | The t distribution is a very important statistical tool. It is used instead of the normal distribution when the true value of the population variance is not known and must be estimated from a sample.  The shape of the t distribution is dependent on the number of points in your dataset. As *n* gets large, the t distribution approaches the normal distribution. For lower values, it has a lower central “hump” and fatter “tails.” |

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| Using the  t distribution in the PSP | In the PSP the t distribution is used in two ways. We use the t distribution to test the significance of a correlation. We also use the t distribution to calculate the prediction interval when using PROBE methods A and B. |

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The t distribution, Continued

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| T distribution function | When numerically integrating the t distribution with Simpson’s rule, use the following function.    where  • *dof* = degrees of freedom  • Γ is the gamma function  The gamma function is  , where  •  • |

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The t distribution, Continued

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| An example of calculating gamma for an integer value | for integer values is . |

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| An example of calculating gamma for a non-integer value |  |

### An example

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| An example | In this example, we’ll calculate the values for the t distribution integral from 0 to *x* =1.1 with 9 degrees of freedom.   1. First we’ll set *num\_seg* = 10 (any even number) 2. W = *x*/*num\_seg* = 1.1/10 = 0.11 3. E = 0.0000001 4. *dof* = 9 5. *x* = 1.1 6. Compute the integral value with the following equation.   where     1. We can solve the first part of the equation:     The intermediate values for this are in the Table 2. |

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| *i* | *x****i*** |  |  |  |  | Multiplier | | Terms |
| 0 | 0 | 1 | 1 |  | 0.38803 | 1 | | 0.01423 |
| 1 | 0.11 | 1.00134 | 0.9933 |  | 0.38544 | 4 | | 0.05653 |
| 2 | 0.22 | 1.00538 | 0.97354 |  | 0.37777 | 2 | | 0.0277 |
| 3 | 0.33 | 1.0121 | 0.94164 |  | 0.36539 | 4 | | 0.05359 |
| 4 | 0.44 | 1.02151 | 0.89905 |  | 0.34886 | 2 | | 0.02558 |
| 5 | 0.55 | 1.03361 | 0.84765 |  | 0.32892 | 4 | | 0.04824 |
| 6 | 0.66 | 1.0484 | 0.78952 |  | 0.30636 | 2 | | 0.02247 |
| 7 | 0.77 | 1.06588 | 0.72688 |  | 0.28205 | 4 | | 0.04137 |
| 8 | 0.88 | 1.08604 | 0.66185 |  | 0.25682 | 2 | | 0.01883 |
| 9 | 0.99 | 1.1089 | 0.5964 |  | 0.23142 | 4 | | 0.03394 |
| 10 | 1.1 | 1.13444 | 0.53221 |  | 0.20652 | 1 | | 0.00757 |
| Result | |  |  |  |  | | 0.3500589 | |

Table 2

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An example, Continued

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| Example, continued | 1. Compute the integral value again, but this time with *num\_seg* = 20. The new result is 0.35005864. 2. We compare the new result to the old result. 4. We can then return the value *p* =0.35005864. |